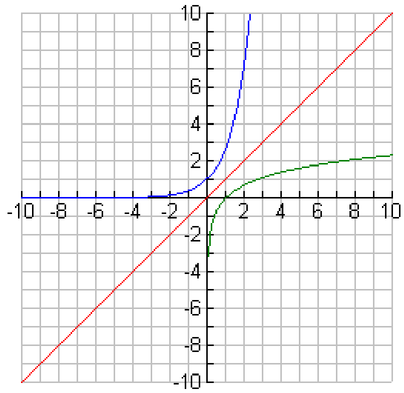
3.3

Logarithmic Functions and Their Graphs

The inverse of an exponential function $f(x) = b^x$ with base b is the logarithmic function with base b, $f^{-1}(x) = \log_b x$.



The domain of the logarithm function is $x > 0$.

Domain of a logarithm function = range of the exponential function = $(0,\infty)$ and the range of a logarithm function = domain of exponential function $(-\infty,\infty)$.

Changing Between Logarithmic and Exponential Form

If $x > 0$ and $0 < b \neq 1$, then **$y = \log_b(x)$ (read as "y is the logarithm to the base b of x")**

if and only if $b^y = x$.

Evaluating Logarithms

See example 1 pg. 300.

Basic Properties of Logarithms

For $0 < b \neq 1$, $x > 0$, and any real number y,

- $\log_b 1 = 0$ because $b^0 = 1$.

- $\log_b b = 1$ because $b^1 = b$.
- $\log_b b^y = y$ because $b^y = b^y$.
- $b^{\log_b x} = x$ because $\log_b x = \log_b x$.

Evaluating Logarithmic and Exponential Expressions

See example 2 pg. 301.

Basic Properties of Common Logarithms

Let x and y be real numbers with x > 0.

$\log 1 = 0$ because $10^0=1$

$\log 10 = 1$ because $10^1 = 10$

$10^{\log x} = x$ because $\log x = \log x$

$\log 10^y = y$ because $10^y = 10^y$

See examples 3, 4, 5 pg. 303.

Basic Properties of Natural Logarithms

Let x and y be real numbers with x > 0

$\ln 1 = 0$ because $e^0 = 1$.

$\ln e = 1$ because $e^1 = e$.

$e^{\ln x} = x$ because $\ln x = \ln x$.

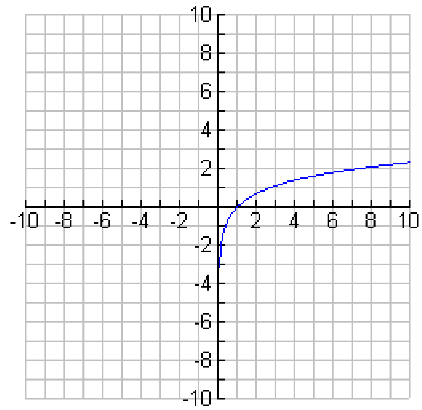
$\ln e^y = y$ because $e^y = e^y$.

See examples 6, 7 pg. 304.

The Natural Logarithmic Function

$f(x) = \ln x$

Domain: $(0, \infty)$



Range: All reals

Continuous on $(0, \infty)$

Increasing on $(0, \infty)$

No symmetry

Not bounded above or below

No local extrema

No horizontal asymptotes

Vertical asymptote: $x = 0$

End Behavior: $\lim_{x\to\infty} \ln x = \infty$

Transforming Logarithmic Graphs

See example 8 pg. 306